Dynamics of Lava Flows

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Abstract. Extensive lava flows, like those of Mount Merapi of Central Java, require thickness only of the order of meters and temperatures only slightly above the melting point to spread over distance of the order of hundreds of meters. Other things being equal the spreading distance is propotional to the cube of the thickness of the flow.

Spectacular The Mount Merapi Of Central Java

The Mount Merapi of Central Java furnishes what is probably the most spectacular display of basaltic lava flows. Not only are there accumulated basalt thickness in excess of the thousands of meters, but subsequent volcanic and giant floods have helped to expose enormous sections of lava flows.

Even a casual observer is, then often amazed to see a particular lava flow that extends over thousands meters without any noticeable variation in thickness and with hardly any indication that the flow was begining to solidify as it was flooding the countryside. It thus seems reasonable to conclude that a gigantic lava lake was established so quickly that lavas did not have time to cool to the point of solidification. Even the direction of the flow is still a matter of speculation.

Since lava flows of such dimensions have never been observed by man, speculation about their dynamics presents an interesting theoritical problem. Probably the most extensive study of the flow rates of flood basalts. However to formulate the equation of motion, we have to take into account the fact that the lavas are cooling off as they flow, and that their viscosity increases rapidly ith a decrease in temperature. The dependence of viscosity μ on temperatur T as being of the form

$$\mu = \mu_0 \exp\frac{b}{T} \quad \text{.................................... (1)}$$

The parameters μ_0 (viscosity at an infinite temperature) and b must be determined experimantally. Recent data by Murace and Mc. Birney (1970) reveal $\mu_0 = 1{,}380 \times 10^{-7}$ kg.m^{-1}.sec^{-2} and b = 2,65 x 10^4 °K. Shaw et al (1986) give slighty higher values of $\mu_0 = 6{,}025 \times 10^{-7}$ kg.m^{-1}.sec^{-2} and b = 2,73 x 10^4 °K (Different parammeters are obtained for melts of basalts, but the difference may be due to later loss of volatiles. Thus, those values are probably less applicable to our problem). Since the Mount Merapi basalts do not exhibit signs of solidification, they must have reached the distant points before their temperature dropped bellow 1150°C. Moreover, they were extruded on substrata that were, on the large scale, horizontal to within about half a degree and probably less. We can thus write the equation of motion of the lava in the greatly simplified form

$$\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - g\beta + \frac{\mu}{\rho}\frac{\partial^2 v}{\partial y^2} \quad \text{..............} (2)$$

Where v is the velocity of the flow, ρ is the lava density, P is the pressure, and β is the slope of the bottom. To simplify calculations, we disregard the inertial terms in (1). Thus, we actually investigate a steady laminer flow rather than a turbulent spreading. But simple reasoning shows that the error thus introduced can result only in a necessary increase in the initial temperature or the thickness. Our result then will yield the minimum necessary lava temperature and thickness.

We further approximate

$$P = \rho.g.h$$

where h is the thickness of the flow. If we assume that the flow velocity is predominantly horizontal, then

$$\rho.g\left(\frac{\partial h}{\partial x} + \beta\right) = \mu.\frac{\partial^2 v}{\partial y^2} \quad \text{.......................} (3)$$

The term $\frac{\partial h}{\partial x}$ is the slope of the upper surface of the flow, to a good approximation can be set equal to a constant $\alpha << 1$. Boundary conditions of vanishing velocity at the bottom of the flow and vanishing shearing stress at its free surface yield

$$v = \frac{\rho.g.(\alpha + \beta)}{2\mu}.y.(y - 2h) \quad \text{................} (4)$$

obviously, to have a positive velocity, $(\alpha + \beta)$ must be negative.

Wether or not solution (4) is justified depends on the Reynolds number R of the flow. The significant linear dimension of the system being h,

$$R \approx \frac{h^3.\rho^2.g.(\alpha + \beta)}{2\mu^2} \quad \text{...........................} (5)$$

with μ of the order of 10^2 poises and $(\alpha + \beta) \approx 10^{-2}$, R will reach the critical value of 10^3 for h of the order of the meters. This coincidence is fortune. It means that turbulance will not make (4) too far for correct; yet, at the same time, there will be enough mixing for lava temperatures to depend primarily on the distance traveled and hardly at all on depth below surface.

Next we analize the process of cooling of the lava (Shaw and Swanson, 1970). Of the three processes of heat loss, conduction is certainly negligible. Large quantities of heat are probably lost by connection in the atmosphere, but numerical estimates (Freagle and Businger, 1963) indicate that unreasonably high wind velocities would be necessary to make this loos comparable to loos due to blackbody radiation. We therefore approximate the loos of heat of the lava layer (Shaw and Swanson, 1970) as being due to radiation alone.

$$-\frac{\partial T}{\partial t} = \left[\frac{\varepsilon.\sigma}{\rho.\eta.a.h}\right] T^4 \quad \text{.......................... (6)}$$

where ε is the specific emissivity of lava, σ is the Stefan-Boltzmann Constant 5,67 x 10^{-8} Joule.m^{-2}.sec^{-1}.°K^{-4}, η is the energy heat equivalent of 4,185 x 10^3 Joule.$Kcal^{-1}$, and a is the specific heat capacity of the lava (approximately 0,2 Kcal.kg^{-1}.°K^{-1}). Heat will also be generated as gravitational energy is clissipated in the internal friction of the lava; heat may also be either generated or consumed owing to entropy changes. The value resulting from gravitation is too small by at least 2 orders of magnitude. The value resulting from entropy changes is not yet known, but it is unlikely that this phenomenon is significant during the early period of spreading.

Since we have better data on the distance floaded x than we have on the duration of flow, we rewrite (6) as

$$-\frac{dT}{T^4} = \frac{e.\sigma}{\rho.\eta.a.h} dx < v > \quad \text{.................... (7)}$$

where $< u >$ is the mean flow velocity

$$< v >= \frac{dx}{dt} = -\frac{\rho.g.(\alpha+\beta)}{3\mu}.h^2 \quad \text{............ (8)}$$

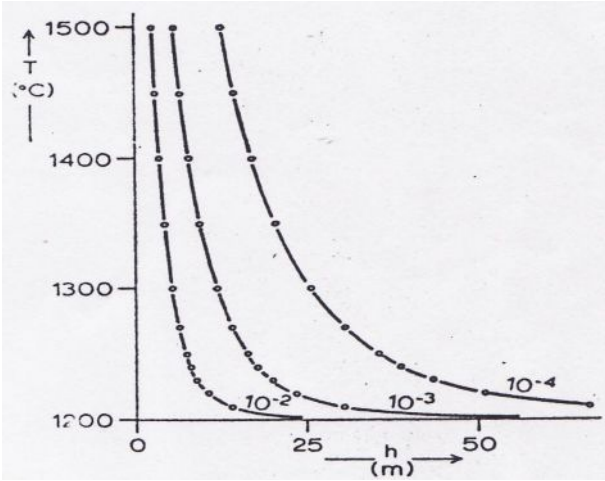
Substituting (1) and (8) in (7), we obtain

$$\frac{dT}{T^4.\exp\left(\frac{b}{T}\right)} = \frac{3.\varepsilon.\sigma.\mu_0}{\rho^2.\eta.a.h^2.g.(\alpha+\beta)} dx \quad \text{....... (9)}$$

This equation can be intregated as

$$\exp\left(\frac{-b}{T}\right)\left[\left(\frac{b}{T}+1\right)^2+1\right]_{T_0}^{T_s} = \frac{3.\varepsilon.\sigma.\mu_0.b^3.x}{\rho^2.\eta.a.h^2.g.(\alpha+\beta)} \quad \text{....... (10)}$$

where T_0 is the initial temperature and T_s is the temperature of solidification (approximately 1150°C = 1423°K)



Minimum lava-flow thicness h and minimum initial temperature T_0 needed to produce a 300 m flow [$(\alpha + \beta)$ as parameter]

Equation (10) was evaluated for ε = 1, and μ_0 and b as derived from Murase and Mc.Birney's (1970) results, x = 300 m, ρ = 2,65 x 10^3 kg.m^{-3} and $(\alpha + \beta)$ = 10^{-2}, 10^{-3}, and 10^{-4}. Results are shown in figure 1. (slightly different curve would be obtained for other values of the parameters. Thus, for ε = 0,5, all values of h would be reduced by a factor of $(0{,}5)^{1/2} \approx 0{,}8$. For values of μ_0 and b derived from shaw et al, 1986, all curves would be sloping to the right at a less steep angle. Nevertheless, the results would, in principle, be the same.

We thus see that lavas can flow enormous distance over nearly horizontal areas, with negligible hydrostatic head, even when their thickness are only of the orders of meters with increasing initial temperature, the necessary flow thickness decreases quite rapidly at first and then decreases asymptotically toward a small but finite value.

Since the cube of h appears in (10), a small increase of the thickness may result in large increases of the distance the flow can travel

All these results are in good agreement with the result of Shaw and Swanson (1970).

An even more gradiose example of such a lava flow may eventually be found on the moon. Since the distance to which the lavas spread there seems to be greater by about a factor of 4, and since gravitational acceleration is lower by a factor of 6, the right hand side of (10) may be greater by about factor of 24. However, since the cube of h appears in (10) the flows thickness needs to be increase by less than a factor of 3 to produce the desired effects.

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